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Teachers learning to design and implement mathematical modelling activities through collaboration

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In this paper, we present a model for upper secondary in-service teacher courses based on the Anthropological Theory of the Didactics and explore how we can teach teachers to design and implement mathematical modelling in their classrooms. The course evolves around Study and Research Path based teaching and strives to create para-didactic infrastructures as a framework for teachers' development of teaching practice. The novelty in this study is the sequence of shared preparation, observation and evaluation of teaching in the course. We describe the structures and their functioning through an example of a group of teachers' work. Based on the activity we discuss the potentials of creating such structures and the needs for further research in this field.

Keywords: Study and research paths, modelling activities, professional development, para-didactic infrastructures, upper secondary education.

Introduction

Throughout the years, several examples of in-service teacher courses on modelling have been presented with the purpose of supporting teachers to design modelling activities, promoting different theoretical approaches to mathematical modelling (Kjeldsen & Blomhøj, 2006; Doerr 2007; Blum & Borromeo Ferri, 2009; Barquero, Bosch, & Romo, 2018). These approaches have ways to engage students in modelling activities, where students sometimes choose strategies different from those foreseen by researchers or teachers. This is challenging for teachers, and might cause them not to implement new knowledge gained from in-service courses, professional development (PD) activities, in their teaching practices. García argues that PD initiatives require existence of structures supporting teachers while implementing more inquiry based teaching methods and he suggests lesson study or other versions of action research (García, 2013). According to Artigue and Blomhøj (2013), mathematical modelling can be regarded as one approach to inquiry-based teaching, which is why those supporting structures, mentioned by García (2013) should be implemented in PDs on mathematical modelling.

In this paper, we present the result of a pilot study, where upper secondary in-service teachers were taught to design mathematical modelling activities based on Study and Research Paths (SRP) from the Anthropological Theory of the Didactic (ATD) including elements of lesson study. García, Higueras, and Bosch (2006), Barquero (2009) and Jessen (2014) have shown how SRP can be used for the design of modelling activities for students at all levels of the educational system. In those studies, modelling functions as a vehicle for learning mathematical content knowledge (see further in Julie & Mudaly, 2007, p. 504). This was also the purpose of the SRPs, developed by the participants of our PD. However, it has been reported that Study and Research Paths for Teacher Education, SRP-TE (Barquero et al., 2018) proved to be difficult for teachers to implement and they turned to transmission of knowledge when back in their own classrooms with designs developed in a PD. In our study, we draw on the suggestion of creating supporting structures for in-service teachers as part of courses on SRP (Muñoz, García, & Fernández, 2018). We adopt the model suggested by Miyakawa and Winsløw (2013), called para-didactic infrastructures, to describe the use of elements of lesson study structures in our PD. These structures cover shared preparation, observation and reflection upon some teaching. Miyakawa and Winsløw (2013) use the model to describe an open lesson observed during a lesson study festival in Japan and to point out why open lesson represents an attractive element of PD for Japanese teachers. We find it interesting to study, how elements of this PD practice can be explored in other contexts. This is the objective of this paper, which address the research question: How can we employ para-didactic infrastructures in PD for upper secondary teachers in order to support teachers' implementation of SRP based teaching in their own classrooms? To answer this, we provide a short introduction to basic notions of ATD, a description of the course and an implemented SRP.

The theoretical framework of ATD – and its use in the course

Praxeology is a core notion in ATD, where we consider it possible to describe all human activities in terms of praxeologies. A praxeology consists of two elements: praxis and logos. If we consider mathematical praxeologies, *praxis* consist of a *type of task* and the *technique(s)* to solve it. An example could be how to find the surface area of an open cylinder with radius *r* and length *l*. The technique to solve it, is the formula $A = 2 \cdot \pi \cdot r \cdot l$. Logos is the justification of praxis and consists of *technology* and *theory*. In the case of the cylinder, the technology is the articulation of how we can cut the cylinder open and unfold it as a rectangle. The length is then the one of the cylinder, *l*, and the width is the circumference of a circle with radius *r*. The theory is a higher level of justification, which in this case will be geometric shapes, their measures and properties. Praxeologies are connected through shared techniques, technology or theory, which form mathematical organisations (MO) from local ones sharing techniques to global ones, describing a whole domain, e.g. vector algebra (Barbé, Bosch, Espinoza, & Gascón, 2005). We can consider teachers' actions in the classroom, as the realisation of didactical praxeologies. What is observable in teaching situations, is often the techniques: the way to introduce new content knowledge, the way to pose questions, the way to organise students' work in groups etc. The didactical tasks, which the techniques answer, are addressed by the teacher when preparing the lesson. The logos of the didactical praxeologies are rarely evident, but might be based on teachers' initial education and courses on learning theory, didactics of mathematics or their teaching experiences. The latter might not count as real theory, but is still a level of justification from the perspective of the teachers. Teaching can then, be considered a set of mathematical and didactical organisations (DO) intertwined and to be realised in the classroom. Miyakawa and Winsløw presents "a theoretical approach to study mathematics teacher knowledge and the conditions for developing it in direct relation to teaching practice" based on ATD (Miyaka & Winsløw, 2013, p. 186), which is depicted in Figure 1. The model illustrates how teacher knowledge can be developed and described in terms of mathematical and didactical praxeologies, relevant for teaching a certain piece of knowledge. The didactic infrastructures refer to the interaction between the MO and DO employed. The PO's represent the paradidactic organisations. The PO_1 is the pre-didactic organisation, including knowledge and practices involved in teachers joined exploration and formulation of the MO to be taught and the DO required to do so. PO_2 is the post-didactical organisation involved in the evaluation of the realised MO and DO. In our PD, we strived to provide teachers with situations in terms of PO_1 and PO_2 and to exploit those, reconsidering the knowledge to be taught, why to teach it and how to teach it – to free the teachers from habits and to construct new knowledge about teaching practices based on SRP.

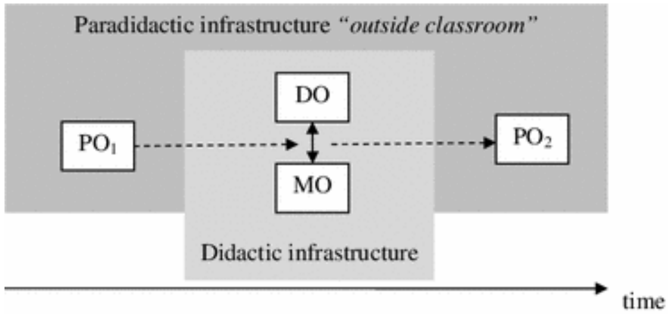

Figure 1: The sketch show the didactic and paradidactic infrastructures surrounding teaching (Miyakawa & Winsløw, 2013, p. 189)

SRP is a design tool suggested to design modelling activities and inquiry based teaching (García et al., 2006). The design of an SRP starts by formulating a generating question, Q_0. Students should be able to understand, but not able to answer Q_0 unless they engage in study and research processes. The study process is when students study different media: textbooks, online webpages, video materials, data from an experiment etc. to gain knowledge on a subdomain, method, formula and more. This process is considered to be the deconstruction of knowledge. In the research process, students combine knowledge acquired in the study process with their existing knowledge forming answers to derived questions (which in the end lead to a coherent answer to Q_0). This reconstruction of knowledge is considered the result of students' interactions with the milieu (Jessen, 2017, p. 224). The dialectic between study and research is assumed to give rise to derived questions. As for the SRP described below, a derived question could be: "in order to answer how long the route is, I need to know how to find the length of a vector?". This question addresses the content, but derived questions can also be technical in their nature, such as how to define a vector in Geogebra? In ATD we consider those questions and their answers as mathematical and instrumented praxeologies. In the planning of a SRP, it might also be relevant to consider meta-cognitive teacher questions such as "what have you done so far in order to answer Q_0". This would be considered part of the didactical praxeologies employed by the teachers. Providing teachers with paradidactic infrastructures, meta-cognitive questions would be addressed in the PO_1, when preparing the lesson discussing whether to pose such a question and how it would affect the students learning outcomes. This shared preparation is supposed to develop their didactical praxeologies. During the PO_2, the teachers' didactical equipment will be further developed, when discussing the learning outcomes of the students in relation to the group of teachers' choices regarding didactical techniques.

A way to share the learning potentials of an SRP, is to map the derived questions and how they are related, which form a mind map like tree-diagram. In ATD these diagrams are created by researchers based on the epistemological analysis of the domain being taught (e.g. see Jessen, 2014). In our PD, the participants were encouraged to develop these maps based on their knowledge on the students' prerequisites, knowledge to be taught, preferred textbooks, possible google hits etc. This is one way to engage the teachers in studying the MO of their teaching, which is the first challenge for teachers engaging with SRP in PD (Barquero et al., 2018). In our PD, the participants were suggested to use the tree-diagrams, named 'knowledge-maps', as navigation tools when teaching (see Jessen & Rasmussen, 2018). When evaluating the teaching design in PO_2, we wanted the teachers to discuss what questions and answers were raised by the students during the teaching, and based on this, discuss the learning outcomes. In ATD research, we use discourse analyses to identify what answers or part of answers students developed and, through those discuss, what questions they might answer. Students raise those implicitly or explicitly. This methodology is fully described in Jessen (2014, 2017). In the PD, the participants did not complete such an a posteriori analysis, but we discussed questions and answers raised by students, as result of the realised MO and DO, which led to suggestions for improving the SRP designs and a revision of the lesson plans. As inspiration for the DOs required to realise an SRP, the participants in our PD were presented with the DOs employed in Jessen (2014, 2017) (group work, sharing sessions, strict time schedules etc.). They were articulated as methods to realise SRPs. The participants were encouraged to develop their own methods, drawing on their teaching experiences. For each group of teachers, the planned teaching was materialised in a lesson plan similar to those developed for the MERIA project (Jessen & Winsløw, 2018, p. 3) with formulations of concrete learning goals, broader goals, age of students, time of school year, type of institution, teaching materials including Q_0, media suggested to the students and the knowledge-map showing the teachers a priori analysis. In the end, we had four columns indicating: timeline, teachers' actions, students' actions and observation notes. During the realisations, the observing group members took field notes on how the lesson differed from the plan and what questions and answers were provided by the students. Depending on, what was possible, the participants in the PD collected testimonial from the lessons in terms of: video recording of students presenting their work, pictures of students' presentations, handed in assignments etc. Inspired by Barquero et al. (2018), we describe the realisation of the PD through one group of teachers' work with a SRP on vector algebra, after providing the context of the PD.

The context of the PD and course design

In 2017 upper secondary education was reformed in Denmark, where mathematics was altered with respect to suggested teaching methods and elements of content knowledge (re)introduced, where others were skipped. This created a need for a PD on didactics related to the specific changes of the content (see elaboration in Jessen & Rasmussen, 2018, p. 346). The course was designed as 7 teaching session lasting 4 hours each. The course covers: the why's and how's of inquiry based teaching and modelling based on SRP as a design tool, piecewise linear functions, vector algebra, discrete mathematics and probability theory. Participants were encouraged to join with a colleague so the teachers could collaborate between sessions, and potentially build para-didactic structures as described in Figure 1. A total of 47 teachers participated (with 1-30 years of teaching experience) from all over Denmark and formed groups of approximately 4 teachers, which they kept working with throughout the course. Every teaching session, except the first one, had an element of sharing and peer-feedback on SRP designs (Q_0, knowledge maps, media, etc.) and on the realised SRPs of each group, based on testimonial from the classroom shared with the rest of the participants. Before ending each session, all groups presented their ideas for the next SRP and got feedback on formulation, feasibility of the lesson plan and further media. The groups then improved their SRPs and lesson plans before realising them in their classrooms. Not all teachers were able to complete or test their SRPs due to extraordinary workloads implementing the reform. But most participants were eager to share and get feedback on their ideas and experiences. The knowledge collected on the PD (from the PO_1 and PO_2) is the teacher's notes (which is also the author of the paper), the finalised lesson plans and the documentation collected and shared by the participants from their implementation of SRPs. Hence, the course does not as such create para-didactic infrastructures around the teachers' practice, but rather it offers the teachers *para-didactic situations* to initiate reflections and professional development.

An example of para-didactic infrastructures from the course

In this section, we will describe an example of how the para-didactic situations were implemented in the PD. A group of teachers worked with the problem of introducing vector algebra in grade 10, which earlier was taught at grade 12. During the third session of the PD, the participants shared teaching materials on vectors. Participants compared materials with the new curriculum and discussed what elements could be captured in a generating question, Q_0. They agreed, that a problem concerning routes and navigation, could create a need for the geometric definition of a vector, which they found interesting. The Q_0 should not require any knowledge from physics, since a great number of the mathematics students do not take physics. This is considered the initiation of the PO_1.

During the fourth session of the PD, the groups shared initial formulations of their Q_0's, they orally explained possible paths or strategies for the students to take, when trying to solve the problem. Our group presented the idea of letting students imagine they were a captain in the Caribbean Sea, who needed to guide his crew from one city to another. The group had found a map and a compass rose and wanted to include Geogebra, but hesitated on how to do this: should they provide students with coordinates or let them draw gridlines on the paper version of the map in order to transfer the route to Geogebra? The choice of media depended on this decision. It was discussed in general at the PD, if students would need vectors or simply use geometry – would suggested media inspire the students to use the notion of vector? It was discussed if the word *vector* should be mentioned in the Q_0 and how that would affect the students' learning. These questions are considered tasks, which might develop teachers' didactical praxeologies and a further enrich the PO_1. The group of teachers completed their SRP design and lesson plan after this session. The learning goals, stated in the lesson plan, for the 120 minutes teaching were: "to know the geometric definition of vectors including position vector, be able to construct a sailing route according to the problem in terms of vectors or linear combination of vectors (incl. being able to add vectors based on their coordinates, be able to scale up vectors), determine the length of a vector and be able to do this in Geogebra". The broader goals of the lesson were: "to gain intuition of vector addition being commutative and associative as well as gain knowledge on unit vectors, see the need for them and deduce one from any given vector". Further the group expected the students to develop problem solving competency and aid and tool competency, while working with the problem. The generating question the group formulated was as follows:

> "Q_0: You are a captain in the Golden Days of piracy in the Caribbean and you are to guide your ship from Havana to Santo Domingo (see the attached map). Your crew covers 'landlubbers', 'treasure hunters' and sailors. They only answer to directions formulated as: "go 20 miles south (S), then 30 miles south east (SE) and then 100 miles North West (NW)". A while ago you made the distance from Aruba to Montserrat in 3 days and you expect to travel by the same average speed. What orders would you give your crew and when do you expect to arrive?"

Figure 2: A screen dump showing the Geogebra file and the compass rose handed out

The students were provided with a Geogebra file where the window looks like the picture on the left side of Figure 2, where unit vectors indicating the directions of N, S, E and W were defined together with points indicating the mentioned cities. Furthermore, students were provided with a compass rose on a piece of paper as the one shown in the right side of figure 2. Media suggested to the students, but not required to use, was: 10 pages in a textbook and a short manual introducing vectors in Geogebra created by the teachers. The group expected students to find media online e.g. Wikipedia and Webmatematik, both in Danish. The lesson was planned to start by showing the students Q_0 and the suggested media. Furthermore, the teacher had added a function to the Geogebra file called “vector from starting point”, forcing Geogebra not to draw all vectors from origin. From the perspective of ATD, we consider the Geogebra file the milieu of the SRP. Students were expected to explore how they can construct a route based on points, lines and vectors depending on how they adapt to the milieu, study the notion of vector and develop an answer in the Geogebra file. In this respect, students are learning from the dialectic between media and milieu, between research and study processes.

The introduction of the problem, media, the Geogebra file, the connection of computers, and dividing the class into groups of three is estimated to take 20 minutes. The students are planned to work on the problem for 20 minutes, where the teachers observe the students and assist them with technical problems in Geogebra. The students present their work during the last 20 minutes. After the lunch break the teacher spends 5 minutes reminding the class of strategies presented earlier and introduce the Geogebra function “vector from starting point”. Then the groups had another 20 minutes preparing before using 20 minutes on students’ second presentations. During the last 15 minutes the students are asked to write down their solutions and strategies with arguments, why they chose the described strategies. After the lesson the students hand in the description together with their Geogebra file. During the following lesson, vector exercises similar to those of the written examinations were worked on by the class. The lesson plan represents the group of teachers’ outcome of the PO_1.

The teachers summarised their experiences and observations and wrote it down straight after the class. Together with the observation notes taken during the lesson, they completed the lesson plan, and from this we get a picture of the initiation of the PO_2. Furthermore, the teachers video recorded the two sharing sessions, where the students connected their computers with the projector and shared their work. During the fifth session of the PD, the teachers presented their experiences with their SRP by sharing the lesson plan, and video recordings. Based on this it was discussed, if the learning goals were achieved and how they were achieved. The videos showed that many students started working with line segments creating a need for directions corresponding to NW, SE etc. This led the students to experiment with the notion of vector and Geogebra syntax. Some students created a compass rose, defining the vector NE as starting from (0,0) and ending in (1,1) and NNE as the vector with end point $(\frac{1}{2},1)$. But the groups found it unpractical with "unit" vectors of different lengths, when calculating the length of the route. Hence, the students needed real unit vectors, which were discussed in relation to the unit circle. In the PD, participants agreed that most students had achieved the intended learning during the two hours. However, it was questioned, if all students were able to solve the tasks of the following lesson, if not being allowed to use Geogebra. What was questioned was the strength of the logos of the developed praxeologies and if those could be used in other contexts e.g. pen and paper mathematics. The peer-feedback and discussion of the realised SRP led to suggestions for improvement of the design and the lesson plan, which concluded the PO_2.

Concluding remarks

From an electronic survey evaluating the PD, we know that the participants felt obliged to prepare, to implement and share experiences because of the structure of the PD. Participants noted that it was the first time they were allowed and encouraged to dwell on and discuss students learning in this detail – and similar for the planning of teaching. This seemed to be sufficient support, making participants comfortable enough to implement their SRP designs in their classrooms. The sharing further encouraged them to realise their SRPs, because of the contagious enthusiasm from those who already tried it. The numbers of realised SRPs indicates, that creating para-didactic situations might further teachers' outcomes of PDs in terms of implementing SRP based modelling in their practice. Still, more research is needed in this area. What is the role of the researcher, teaching the PD? How does the PD affect teaching practices in the long run? Are the collaborations between participants sustainable and under what conditions? And what research methodologies can capture the MO and DOs developed by students and participating teachers respectively in large-scale studies?

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